NAVAL REACTORS PHYSICS HANDBOOK

VOLUME I

Selected Basic Techniques

Volume Editor

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Chapter 1

REACTOR PHYSICS AND ITS APPLICATION TO NUCLEAR POWER REACTORS

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1.1 INTRODUCTION

From the inception of the Naval Reactors and Shippingport programs major effort has been devoted to reactor physics and associated technology. The work in this area has been motivated by the need for the design of safe, reliable, and efficient reactors. In carrying out these reactor physics programs, available techniques have been modified and extended, and new techniques have been developed. The latter can always be traced back to basic problems associated with the accurate predictability of the performance of particular reactors. The problems have originated partly from the appearance of new reactor concepts, partly from the demands for the design of reactors having higher performance and other improved characteristics. It is the purpose of this book to describe a number of the reactor physics techniques that have proved useful. It is hoped that this information will be of assistance to the reactor physicist and nuclear engineer in exemplifying the application of physics to the practical problems of reactor design and analysis. There has been no attempt to make a complete treatment of reactor physics nor, on the other hand, to write a design manual. Rather, each discussion treats a particular problem in reactor physics, presenting approaches to the solution of these problems that have been profitable. As pointed out in the Foreword, this volume emphasizes basic and generalized approaches, while Vols. II and III [1,2] cover applications to specific designs.

When the Naval Reactors program was initiated the only cores of significant power output which had been built were of the natural uranium graphite type. These cores had characteristics lending themselves to relatively simple physics procedures. Specifically, the cores were macroscopically homogeneous, the spectrum of neutrons causing fission was well thermalized, leakage out of the core was low, control rods were of circular shape, and core endurance was so short that depletion effects, other than those due to transient fission product poisons, could be treated as perturbations. As a result, treatments based on age diffusion theory were adequate for the description of such cores. All calculations were performed on desk calculators.

At about this time several reactor design efforts of particular significance for the Naval Reactor program were begun. One of these, the power breeder program at the Knolls Atomic Power Laboratory (KAPL), consisted of reactor studies partly or primarily in the neutron intermediate energy spectrum. The KAPL program was subsequently reoriented towards the design of a naval reactor operating in the intermediate spectrum. Another design effort begun about this time was that of a small light-water power reactor, the prototype of the first Naval Reactor (STR). The work originated at the Oak Ridge National Laboratory and was transferred to the Argonne National Laboratory. Subsequent STR design work was carried out at the Bettis Atomic Power Laboratory. The development of light-water reactors for the Naval Reactors and Shippingport programs has continued at Bettis, KAPL, and the Naval Reactors Division of Combustion Engineering Corporation.

In each of these development programs of high performance cores the reactor properties have determined the direction and emphasis of the physics efforts.

For cores of the types studied at KAPL and elsewhere, in which fissions occurred principally at intermediate neutron energies, neutron multigroup analysis was developed. A basic treatment of multigroup diffusion theory is given in Chap. 2 of this volume, and Vol. III of this handbook gives a detailed account of the physics of intermediate energy spectrum reactors. While the Naval Reactors program at the present time utilizes light-water cooling and moderation, the fuel loading densities are often sufficiently high that a substantial fraction of the fissions occurs above thermal neutron energies. Hence, much of the physics work required for describing the intermediate energy spectrum is useful for light-water reactors. Advances in the following areas have been of particular benefit:

Statistical theory of resonances of fission products (Chap. 4).
Theory and direct measurement of Doppler broadening of resonances (Chap. 2 and Vol. III).
Interpretation of adjoints and calculation of reactivity coefficients (Chap. 5).
Self-shielding of resonances (Chaps. 2 and 4).
Critical assembly experimentation (Chap. 6).

1.2 REACTOR PHYSICS AND NUCLEAR DESIGN OF NAVAL REACTORS

In early cores of low fuel and poison loading, the high moderating power of light water produced a neutron spectrum quite closely thermal, and a two-group (or at most a four-group) treatment was considered appropriate. However, a major problem was presented as to the proper group parameters to be used; great effort has gone into the solution of this problem. Chapter 2 of this volume treats the calculation of parameters for fast neutrons, and Chap. 3 for thermal neutrons.

In the case of the fast group it was known that the continuous slowing-down model in use for heavy moderators would be inaccurate for hydrogen moderators. More suitable models for hydrogen slowing down were utilized. This work was supplemented by experimental determinations of the slowing-down distribution in light-water metal mixtures. A second difficulty was that, as a result of the high leakage in the cores being designed, the fast neutron spectrum was quite different from that in an infinite medium. In this case, since the fast neutron spectrum does not vary rapidly from point to point in the core, it is a good approximation for the purpose of computing parameters to assume a sinusoidal space dependence of the flux with a geometric buckling equal to that of the average material buckling in the region of interest. The multigroup treatment then becomes mathematically the same as the normal mode spectrum calculation for an infinite medium. The calculation yields the neutron spectrum appropriate to the region and, by averaging the energy-dependent cross sections over the spectrum, the fast group parameters can be obtained.

The importance of inelastic scattering by heavy elements in slowing down was recognized early, but accurate treatments were held back by lack of experimental data. In recent years a great many measurements have been made and, furthermore, an extensive program for calculating such cross sections has been developed. The calculation program is covered in Chap. 2 for elements of particular interest to the Naval Reactors program.

In the thermal energy region a formidable problem was presented by the rapid change in hydrogen binding and cross section with neutron energy. The Radkowsky prescription furnished a phenomenological method of obtaining the thermal transport mean free path from the measured variation of the hydrogen cross section with neutron energy. Since these core types had much greater thermal macroscopic absorption than the natural uranium graphite cores, it was necessary to take into account the deviation of the thermal spectrum from a

Maxwell distribution due to absorptions occurring before the neutrons could come into thermal equilibrium with the moderator. This was accomplished by application of the Wigner-Wilkins formalism and improved models.

Because of the small slowing-down length in light-water cores and the large amounts of reactivity associated with transient poisons (xenon and samarium) and temperature change, control requirements in these cores were relatively stringent and necessitated a close spacing of control rods. An associated problem was that the small thermal diffusion lengths resulted in large flux distortions due to heterogeneities, such as control rod water channels. Thus, to avoid excessive water-hole peaking it was necessary to utilize control rods consisting of thin plates in a relatively complex geometry, for example, cruciform. The overall result was a core which was quite complex from a physics standpoint. An accurate description of the flux peaks and the control rod worth requires an explicit representation of the control rods, structural members, water regions, fuel regions, and other features of the core affecting the neutron distribution. To provide such a description it has been necessary to carry on investigations in such areas as self-shielding and cell theory (Chap. 4) and blackness theory (Chap. 3), and to make full use of the best mathematical tools as they have become available, including successively desk calculators, small digital computers, manually adjusted electric network simulators, and large scale digital computers ultimately leading to large two- and three-dimensional diffusion theory computer programs (Chap. 7).

In addition to the development of methods for the static description of the cores, a great amount of work has been required to obtain suitable treatments of kinetics, safety, and stability problems (Chap. 5). The characteristics of light-water cores which are significant from a kinetics standpoint differ in many essential aspects from those of cores previously studied: for example, the light-water cores have much larger moderator and void negative temperature coefficients; much smaller metal reactivity coefficients; much smaller thermal neutron lifetimes; different potential accidents, such as the cold water accident; and different transient behavior occasioned by the need for rapid and flexible maneuvering.

In the Shippingport program, as an approach to economic power generation, the emphasis has been on use of fuels of high U^{238} content in order to minimize diffusion plant costs. This has led to the study of light-water uranium lattices which, owing to the relatively close spacing of the fuel elements, have unique characteristics: large interactions, particularly in U^{238} fission (fast effect) and in resonance absorption (Dancoff effect), and a large contribution to slowing down by the uranium inelastic scattering. To understand and describe such lattices, a joint program was set up between the Bettis Atomic Power Laboratory and the Brookhaven National Laboratory in which sets of slightly enriched uranium fuel rods were studied, alternately, at Brookhaven in a subcritical or exponential assembly and at Bettis in a critical assembly. Some of the associated techniques are described in Chaps. 2 and 6 of this volume, and in Vol. II.[1] A new phase in the program began with the investigation of seed-blanket cores, which differed conceptually from the types of cores considered previously. The basic idea of the coupling of two quite dissimilar core regions, or of driving one core region by another, led to a deeper insight into the meaning and measurement of reactivity. It was necessary to treat such novel constructs as the extremely high neutron flux gradients existing between the seed and blanket regions, the seed-blanket power sharing, and the concept of a large core having essentially the kinetics properties of a small core (the seed). Furthermore, the depletion characteristics of these cores necessitated study of the properties of plutonium isotopes in light water and of fission products in very high burnup fuels in which most of the energy is derived from the fissioning of plutonium.

The advent of burnable poisons had a major effect on light-water reactor designs. In the first place, the much greater core endurances made possible by the use of burnable poisons resulted in depletions so large that the core parameters changed greatly throughout life. This made it necessary to examine the adequacy of the core from reactivity and thermal standpoints at many times during core life, so that in effect the reactor designer was faced with the problem of designing many cores. As a result of the initial neutron flux spatial variations, the cores, even if they were homogeneous to begin with, became heterogeneous both radially and axially after appreciable depletion. This made it imperative to develop codes to provide two- and even three-dimensional descriptions of the core, not only initially, but throughout life as well (Chaps. 4 and 7). The previously used rough approximation of stable fission product absorption as a constant number of thermal barns per fission was quite inadequate for accurate calculations of endurances with such high fuel depletions. It was found necessary to go to a detailed study of the data on individual fission products, taking into account successive transmutations and epithermal absorptions.

The great increase in fuel loading concomitant with the increase in endurances resulted in blacker cores, i.e., cores with much larger macroscopic thermal absorption cross sections. This, in turn, necessitated a much more careful examination of neutron thermalization and the spatial variation of the neutron spectrum. The powerful variational technique was found to have many applications in the field. The extended use of burnable poisons led to their utilization outside the fuel, in lumped self-shielded form, both macroscopically (e.g., plates and rods) and microscopically (particles). This again introduced the need for extensive calculations and experiments to determine accurately the self-shielding of the poisons and, in some cases, of the fuel, both initially and as a function of fuel depletion. In many cases, conventional analytic methods have been inadequate and it has been necessary to utilize Monte Carlo techniques (Chaps. 3, 4, and 7).

The desire to obtain optimum reactor performance has also resulted in more complex designs. Local and gross zoning of fuel and burnable poisons has been studied to maintain power distributions of specified shapes and to minimize the number of mechanically moving control elements. In this connection, it is of interest to note that, as a core fuel loading increases, the emphasis of the problems changes. For example, in lightly loaded cores the reactivity associated with transient xenon and samarium is very large; secondly, it is often difficult to obtain a large, negative temperature coefficient of reactivity. On the other hand, in cores having a high fuel loading the xenon and samarium reactivity effect is small and the temperature coefficient large and negative, but major problems are presented in power peaking near geometric discontinuities and in shutting the core down with adequate margin. It is also noteworthy that the designer must aim at a balanced core design since changes in the core to improve one characteristic may adversely affect another. Thus, improvement of the power distribution often leads to a reduction in the effectiveness of a given set of control elements.

1.3 NUMERICAL ANALYSIS AND DIGITAL COMPUTER PROGRAMS

To provide detailed descriptions of these complex and advanced designs throughout core life, substantial effort has been placed on numerical analysis. Chapter 7 discusses some of the progress made in that effort, particularly in the treatment of the few-group two-dimensional nuclear diffusion equations, the transport equations, and Monte Carlo methods. Faster convergence rates in the numerical solutions have been obtained over the years by utilizing more sophisticated over-relaxation techniques. In addition, a more rigorous foundation for the basic numerical processes has been established.

This work in numerical analysis stimulated fundamental mathematical investigations in the nature of the spectrum of the multigroup diffusion operator and finite difference approximations to it. The results provided a mathematical basis for the concepts of criticality, multiplication factor, principal distribution, and importance function in nuclear reactor theory. The existence of expansions, similar to those in quantum mechanics, of the solution of the diffusion problems in terms of the characteristic functions associated with the diffusion operator was established.[3] This, again, provided a firm basis for handling many problems in reactor theory, e.g., involving perturbation theory and pulsed source techniques.

As a result of the extensive theoretical analysis advantages have been gained in computer utilization which can be exploited either for reduced design time or greater design capability. Some measure of the gain can be indicated by reviewing the effects of numerical analysis studies on successive programs of the PDQ (two-dimensional diffusion theory treatment) series of digital programs. QED-1 was the first two-dimensional few-group program for the IBM-704. Its successor, PDQ-1, gained in speed as well as convenience to the user by the insertion of automatically calculated overrelaxation factors in the iterative method. The time savings have varied with the experience and ability of the designer. However, since the convergence rate is very sensitive to the choice of overrelaxation factor, the improvement is substantial. PDQ-2 utilized Chebyshev polynomials to carry out the numerical method of solution and was estimated to decrease the overall time by a factor of 1.6. PDQ-3 utilized single line rather than point overrelaxation and increased the speed of a given problem by an average factor of 1.4 over PDQ-2 (some design problems actually showed improvements of a factor of 5). Finally, PDQ-4, produced for the Philco-2000, further decreased the average time by a factor of 1.4 by using double line overrelaxation (not including the increase in speed provided by the machine).

In addition to the improvements in iterative processes, improvements in the iteration strategy, i.e., the choice of the number of iterations performed in a single group before moving to the next group, provided an increase in the overall convergence rate. Such advances have, for example, been incorporated in KARE with the Peaceman-Rachford iteration procedure. Insertion of symmetry conditions of various kinds for half core, quarter core, and eighth core periodicity also introduced substantial factors of gain in speed for a specific problem. However, probably the most important result of the work in numerical analysis has been the development of an understanding of, and a sound theoretical foundation for, the mathematical processes involved, providing insight into the approaches which should be taken for specific problems and the accuracy which can be expected (Chap. 7). Reference 4 is a review paper of this work and contains a rather complete bibliography for further reference.

The need for an accurate and detailed description of reactor behavior has had a marked effect on the development and verification of calculational tools used by the nuclear engineer in the design effort. The nuclear designer is faced with making a compromise between the accuracy and detail with which he desires to describe a reactor and the cost and limitations of computer machinery available to him. Since practical considerations dictate that extensive approximations be made in the techniques used to obtain design information, dependence on more basic calculations and comparisons with experiment for evaluating the accuracy and deficiencies of the design tools have been essential. For these reasons, the formulation and use of more exact calculations have been very important. The studies made in this area, in addition to yielding standards to which more approximate calculations may be compared, have supplied insight into the reactor physics phenomena of interest and guided the formulation of more approximate descriptions that retain the important features of the theory. The use of the more basic theory itself has also identified many areas of sensitivity and guided the reactor physicist in further studies.

While it is possible to develop definitive techniques in reactor theory, they are generally available only in a restricted sense. They may incorporate a detailed description of the energy behavior or the angular dependence but be limited in the spatial description available. For instance, the use of multigroup Monte Carlo has been extensive, but the tractable application of this technique has been limited to small cells in either the resonance region or the thermal neutron range.

1.4 SELECTION AND VERIFICATION OF CALCULATIONAL MODELS

While these basic techniques are accurate in their sphere of application, they are not general enough to obtain all the information about the reactor behavior that is desired. The essentials of these different techniques must be brought together in a self-contained nuclear design model that will describe the entire configuration. This usually requires drastic approximations in all of the variables. Practical examples of this are the use of few-group diffusion theory, performing spatial calculations at discrete points rather than continuously, representing a group of fuel plates and water channels as a homogeneous region, and restricting the calculations to a few selected times during the core lifetime. Identifying this framework does not specify the calculational model but only outlines the ground rules within which the model will be formulated. The few-group cross sections, transport corrections, geometrical detail required, and spatial mesh necessary for the adequate description of flux shapes and depletion effects must be specified in such a manner that the essential features of the configuration are described accurately. This will often be supplemented by including self-shielding factors (for particles, plates, resonances, etc.) in the few-group parameters, using blackness theory to describe the effect of strong absorbers, or by adjusting the few-group parameters to get agreement with a more exact calculation. The basic theory is then used to evaluate the accuracy of the approximations and to identify the required improvements.

While many of the features of the model may be tested against standards, these comparisons must usually be made in a restricted sense, while other features are less susceptible to detailed comparison. Coupled with this is the lack of complete and accurate nuclear data with which to perform the calculations. The comparison and testing of the derived calculational model (using the best nuclear data available) with experimental measurements on critical reactor configurations are, thus, an essential step in further evaluating the limitations of the model. Especially important have been the clean critical assemblies which provide a simplified geometry especially designed to test particular aspects of the theory (Chap. 6). Originally, the experiments were carried out at room temperature. Since the parameters of light-water cores change appreciably from room to operating temperatures, pressurized critical facilities have been provided to permit studying the nuclear assemblies over a wide temperature range. In addition, the use of pulsed source techniques has permitted the study of the reactivities of subcritical assemblies. This has allowed the calculations to be checked at many points rather than merely at critical and has yielded reactor physics information from smaller amounts of material and smaller experimental effort.

The adequacy of the model for use in design problems is still dependent on the degree of detail and accuracy required by the design objectives. If these are extensive, the final stage of design must be paralleled with the testing of the model against a detailed mockup of the core. The amount of information required for design of the core does not allow the obtaining of all this information experimentally. Nor will it yield information on lifetime performance, but a careful analysis of the mockup will supply a valuable normalization point and final verification of the ability of the design model to describe the core configuration. Finally, prototype cores have been heavily instrumented and extensive test programs conducted to verify the design calculations.

1.5 DEPENDENCE OF DESIGN DETAIL ON PERFORMANCE REQUIREMENTS

It may be fitting to close this chapter with a concrete example of the necessity for extending reactor physics calculations to greater detail as a result of increased core performance requirements. In early reactor designs only knowledge of the overall neutron flux and resultant power distributions was essential. As higher power densities were demanded, it was important to determine what has been called by analogy with quantum mechanics the fine structure, i.e., the power output of each individual fuel element. Finally, it has developed in the design of the second Shippingport core, as well as in other high performance cores, that it is necessary to go even beyond this point to what might be called the hyperfine structure, i.e., to the determination of the power distribution within a fuel element. In the second Shippingport core the design maximum power density in the seed is almost 500 watts/cc, more than twice that of the first Shippingport core. As shown in Fig. 1.1 each seed subassembly contains highly enriched oxide fuel wafers, 0.250 in. wide, embedded in zirconium plates. In order to reduce the power peaking, a zoning of each seed subassembly in 2 zones of fuel was adopted, based upon the calculated peak-to-average value of the hottest wafer in any zone to the average in the subassembly. This gave a calculated value for the peak-to-average of about 1.4 which was in reasonable agreement with experiment. However, examination of the power distribution indicated that there was a sharp gradient in the vicinity of the hottest wafer. A special experiment was devised to study this gradient, using four quarter-width wafers in place of the hottest wafer.

The induced gamma ray fission product activity of each quarter wafer was measured and an 18 percent rise in power found across the quarter-inch width of the wafer (Fig. 1.2). As a result of this unanticipated peaking, it was estimated that the thermal output of the core would be reduced by 10 percent. To rectify the situation a new zoning scheme was adopted (Fig. 1.3) with 3 fuel zones. Figures 1.4 and 1.5 compare the power peaks as a function of seed fuel depletion for the original and the new zoning schemes. This is an example in which refinement of reactor physics studies has led to a significant economic benefit in plant performance.

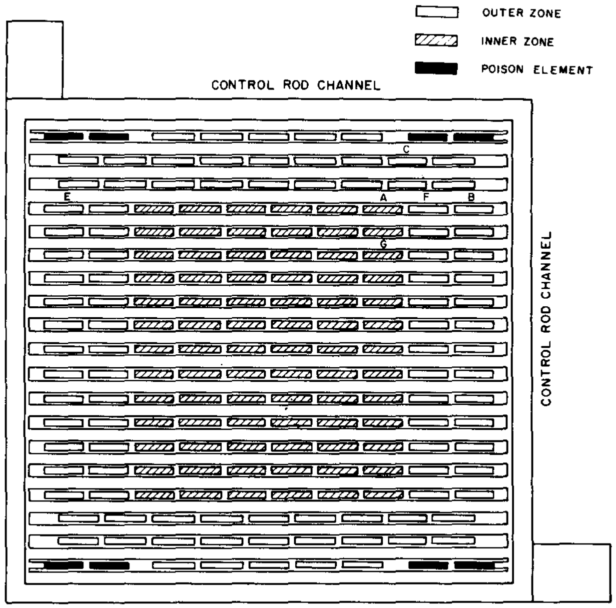

FIGURE 1.1. Shippingport Core 2 Seed 1 Original Seed Zoning.

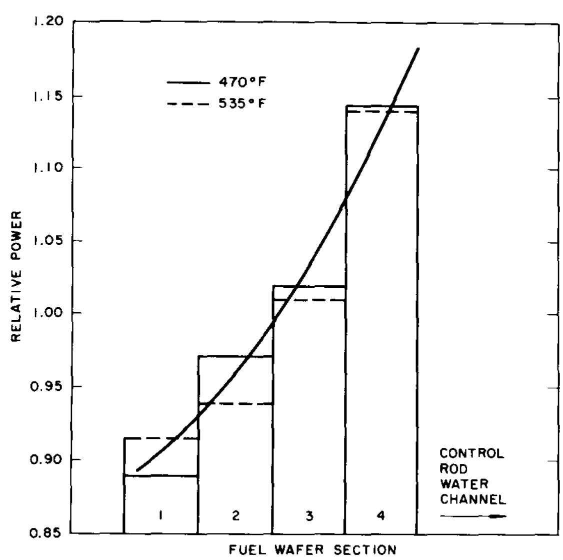

FIGURE 1.2. Shippingport Core 2 Seed 1 Distribution of Power within a Seed Fuel Wafer Located Adjacent to a Control Rod Water Channel.

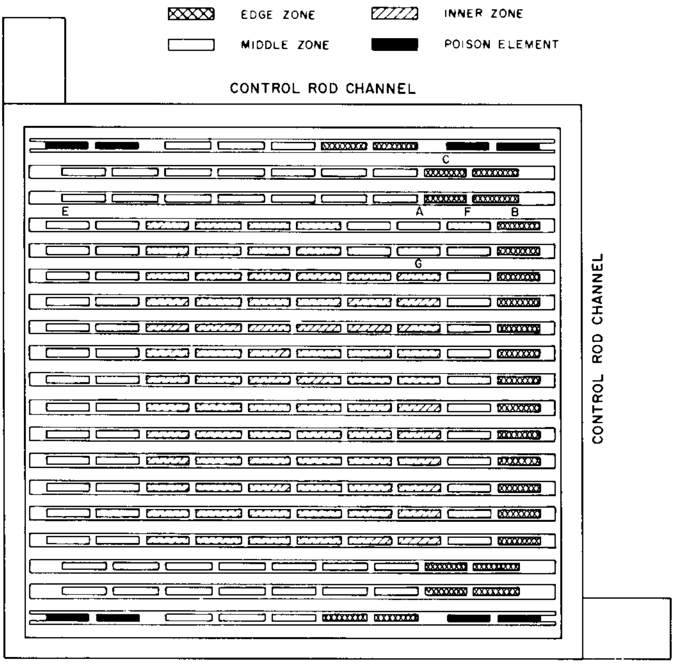

FIGURE 1.3. Shippingport Core 2 Seed 1, New Zoning of Seed to Optimize Power Distributions, Taking into Account Intrawafer Flux Gradients.

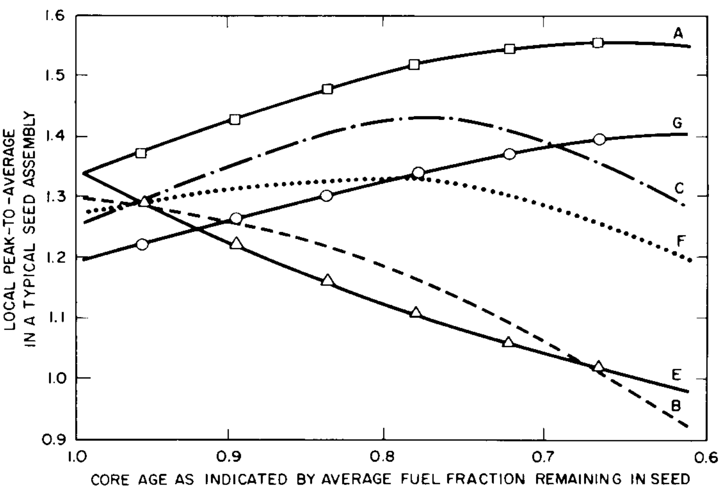

FIGURE 1.4. Shippingport Core 2 Seed 1, Peaking Factor Lifetime Variation as Calculated for Original Zoning (A, B, C, E, F, and G represent various points within the fuel assembly shown in Fig. 1.1).

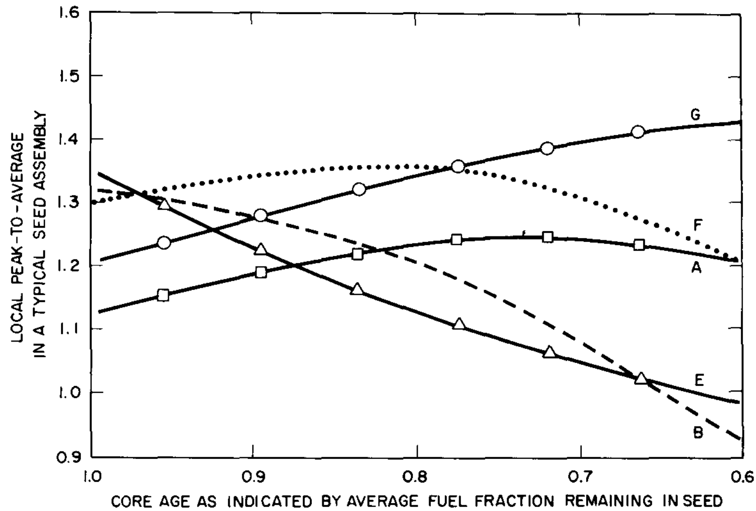

FIGURE 1.5. Shippingport Core 2 Seed 1, Peaking Factor Lifetime Variation as Calculated for New Zoning (A, B, E, F, and G represent various points within the fuel assembly shown in Fig. 1.3).

1.6 SUMMARY

The reactor physics work in the Naval Reactors and Shippingport programs has been guided by the objective of a detailed description of complex reactors designed for optimum performance. The need for accurate calculational and experimental techniques to meet design requirements has required the development of systematic procedures and appropriate tests of their adequacy. The following chapters describe a number of useful techniques and procedures that have evolved in the course of this program.

Finally, it should be pointed out that, as is inevitable in works of this kind, some very recent developments could not be included in this volume. As the work of the Naval Reactors and Shippingport programs continues to advance at a rapid pace into areas of great practical importance as well as of inherent physics interest, the editor must establish a cutoff point in time if a book is to be published.

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